Study on the Impact of Spectral Risk Measures on Optimizing VaR Parameter Selection

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ABSTRACT

Spectral risk measures (SRM) is an advanced financial risk measurement tool, which quantifies the possible losses of portfolio by assigning different weights to the losses of different quantiles. This paper studies how spectral risk measures affect the optimization of VaR parameters to better capture tail risk. The data used is the daily return of CSI 300 index from 2020 to 2024. The analysis process includes data processing, parameter optimization, model verification and comparative analysis. The results show that: (1) Using spectral risk measures can significantly improve the ability of VaR to capture tail risks. (2) By introducing risk aversion function and adapting to different risk preferences, spectral risk measures solves the core defect of traditional VaR in capturing tail risks.

Keywords: Spectral Risk Measures; VaR Parameter Selection; Parameter Optimization; Risk Measurement

1. INTRODUCTION

With the deepening of global financial market reform and more and more extreme risk events, the price fluctuation and risk status of global financial assets have been greatly affected. As the most commonly used risk measurement tool in the financial industry at present, Value at Risk (VaR) is also the core risk indicator recommended by Basel Accords, and has become an important pillar for our financial institutions to allocate venture capital and conduct portfolio management. [1] The existing research has confirmed that choosing the confidence level and holding period of VaR needs to balance the risk tolerance, regulatory requirements and liquidity matching principle [2]. This field is gradually shifting from empirical judgment to quantitative optimization. By introducing heavy-tailed distribution, volatility clustering model and correlation capture tools, the parameters are more in line with the actual market characteristics [3]. However, these studies are mainly aimed at traditional risk measurement, and there is still a lack of comprehensive models that can capture extreme risks and express risk preferences at the same time.

When the traditional VaR parameter selection can't solve subjectivity, enhance tail risk capture or meet the requirements of risk preference, spectral risk measures integrate risk aversion function into risk preference, and at the same time fully capture tail risk of financial assets. This method retains the intuition of VaR, and at the same time makes up for its shortcomings in risk preference description and tail risk measurement. Although the existing research has confirmed the effectiveness of spectral risk measures in incorporating risk preference and enhancing tail risk sensitivity, and further expanded its application in portfolio optimization and extreme risk measurement [4]. those researches on combining spectral risk measures with VaR lack a systematic model to optimize VaR parameters under spectral risk measures. They do not fully consider the complexity of selecting parameters under the restriction of multiple financial markets, and the empirical research is still weak and lacks universality. Therefore, the effect and advantages of optimizing parameters are hard to be strongly supported, which leads to the limited practicability and generalization of the research results.

Therefore, this paper constructs a system optimization model, which combines spectral risk measures and VaR parameter selection optimization depth. By considering the distribution characteristics of financial assets and different risk preferences, we made an empirical analysis to prove that this model is effective. In this way, some problems of traditional VaR parameter selection are solved, such as being too subjective, not paying enough attention to the tail risk, and not matching the actual risk preference. The theoretical significance of this study lies in enriching the theoretical framework of VaR parameter selection and breaking through the limitations of traditional parameter selection. Its practical significance is to provide a more accurate and reliable risk measurement method for financial institutions, which can reduce possible losses in the face of financial risks and improve the efficiency of enterprise risk management.

2. THEORETICAL FOUNDATION

2.1 Definition and Calculation Methods of Value at Risk (VaR)

VaR, also called Value at Risk, refers to the maximum loss that a portfolio may suffer in a specific time period and at a given confidence level under normal market fluctuations. Expressed by mathematical formula, P [x > var] = 1-□, where X represents the loss amount of financial assets in a certain holding period, and α represents a specific confidence level. The higher the α value, the greater the VaR value, indicating the greater the potential loss. The main calculation methods of VaR include variance-covariance (variance-covariance method), historical simulation (historical simulation method) and Monte Carlo simulation (Monte Carlo simulation method). Considering the "fat tail" and peak distribution characteristics of financial asset returns, this paper mainly uses historical simulation (historical simulation method) to calculate. This method does not need to assume that the data conforms to the normal distribution, but directly captures the fat tail effect and nonlinear risk with real historical data, which can reflect the direct profit and loss of financial assets more intuitively. [5] The specific steps are as follows: (1) Collect Historical prices of financial assets and calculate daily income. (2) Reorder the calculated historical income from small to large. (3) According to the confidence level α, find the corresponding quantile k = (1-α) ×n, where n is the sample size of historical data. (4) The k-th income value in the sorted sequence is the VaR value.

2.2 Definition and Calculation Method of Spectral Risk Measure

Spectral risk measurement is a risk index, which is obtained by weighting the quantile of loss distribution (such as VaR) with the risk aversion function φ(q). It is defined as:

$$\mathrm{SRM}(X) = \int_0^1 \mathrm{VaR}_\alpha(X)\phi(\alpha)d\alpha \tag{1}$$

where $\mathrm{VaR}_\alpha(X)$ denotes the Value at Risk value at confidence level α, and $\phi(\alpha)$ is the risk aversion function satisfying $\phi(\alpha)\geq 0$ and $\int_0^1 \phi(\alpha)d\alpha=1$. For the selection of the risk aversion function, this paper adopts the exponential risk aversion function $\phi(p) = \frac{\frac{1}{\lambda}e^{-\frac{1-\alpha}{\lambda}}}{1-e^{-\frac{1}{\lambda}}}$, where $\lambda>0$ is the risk aversion coefficient. A smaller λ indicates greater risk aversion, assigning higher weights to tail losses; a larger λ indicates lesser risk aversion, resulting in a more uniform weight distribution. This paper primarily selects three scenarios: λ=0.1 (high risk aversion), λ=0.5 (medium risk aversion), and λ=1.0 (low risk aversion), accommodating the risk preferences of different financial institutions. The specific calculation steps are as follows: (1) Calculate $\mathrm{VaR}_\alpha(X)$ values for different confidence levels α and sort them from largest loss to largest gain. (2) Use the risk aversion function to compute the corresponding weights for each confidence level. (3) Multiply each $\mathrm{VaR}_\alpha(X)$ by its corresponding weight and sum the results to obtain the spectral risk measure value.

3. EMPIRICAL DESIGN

This paper wants to establish a parameter selection model that can minimize spectral risk measures. The specific model is built like this:

3.1 Function Selection

We choose spectral risk minimization as the calculation function, and its expression is: minsrm (x) = ∫ _ 0.1VAR _ α (x) (α) dα, where α represents the confidence level, VaR_α (X) is the corresponding VaR value of each confidence level, and φ (α) is the risk aversion function.

3.2 Constraints

Combining the regulatory requirements of Basel Accord, statistical validity and the operational logic of financial markets, we have set the following rules:(1) Confidence level range: 0.95 ⩽ α ⩽ 0.99, and take a value every 0.005, namely α ∈ {0.95,0.955, ...,0.99}, which can ensure compliance with Basel's regulatory requirements and daily risk monitoring.(2) Holding period range: t ∈ {1, 2, ..., 10}, which can meet the liquidity needs of short-term (1-3 trading days), medium-term (4-7 trading days) and long-term (8-10 trading days) assets.(3) Risk preference constraint: λ ∈ {0.1, 0.5, 1.0}, which can adapt to financial institutions with high, medium and low risk aversion.(4) Back-testing requirements: the failure rate corresponding to the optimized VaR parameters (that is, the ratio of times the actual loss exceeds VaR) must satisfy

$|f-(1-\alpha)| \leqslant 0.02$, where f is the actual failure rate. This can prevent us from underestimating risks and avoiding excessive risk avoidance.

3.3 Model Processing

This paper uses grid search and particle swarm optimization algorithm to deal with the model. This can not only ensure the calculation efficiency, but also find the best parameter settings, and make the back-test results more stable [6]. In order to strike a balance between computational efficiency and search accuracy, let the model have enough evolutionary space to accurately find the minimum spectral risk measure, and at the same time maintain the diversity of the population to avoid falling into the local optimal solution, we set the number of particles to 50 and the number of iterations to 500. The specific steps are as follows: (1) We need to build a three-dimensional lattice, where α has 9 values, T has 10 values and λ has 3 values, so there are $9\times10\times3=270$ different parameter combinations. (2) Then, we need to calculate a spectral risk measure and a backtesting failure rate for each parameter combination. Remove all the combinations that do not satisfy $|f-(1-\alpha)| \leqslant 0.02$. (3) In the remaining combinations, we select the parameter combination with the smallest spectral risk measure value. (4) Use this selected parameter combination as the starting point of particle swarm optimization algorithm. 50 "particles" are randomly generated around it, and the SRM value of each particle is calculated. (5) After 500 iterations, we constantly update and shrink the smallest SRM value in all particles, so that we can find the final and smallest SRM point very accurately and make our model more accurate.

4. EMPIRICAL RESEARCH

4.1 Sample Selection

This study uses CSI 300 Index as our sample, and uses the daily closing price from January 1, 2020 to December 31, 2024. CSI 300 contains 300 stocks with the largest weight and the best liquidity in the A-share market, which can represent the total value of a large part of the market. The period we have chosen includes several important market stages: for example, large fluctuations in early 2020, high shocks in 2021, and structural adjustments from 2022 to 2023. It also covers some important external events, such as geopolitical conflicts and aggressive interest rate hikes of Federal Reserve, so it is very suitable to test the robustness of our model in extreme cases.

4.2 Empirical Analysis

The method used in this study is: processing data-optimizing parameters-verifying models-comparative analysis. The specific steps are as follows:

(1) We evaluate the distribution characteristics of daily income data by calculating skewness, kurtosis and Jarque-Bera statistics of CSI 300 Index daily income series. Judge whether it conforms to the normal distribution, and verify whether the historical simulation method is feasible. (2) Using the VaR parameter selection model based on minimizing the spectral risk measure and the model processing method, the spectral risk measure value of each parameter combination is calculated. Select the parameter combination with the smallest spectral risk measure. (3) Conduct backtesting on the optimized parameter combination. This study adopts the Kupiec failure rate test by calculating the likelihood ratio (LR) statistic: $LR = -2\ln\left[\frac{(1-p)^{n-x}p^{x}}{(1-f)^{n-x}f^{x}}\right]$, where n denotes sample size, x represents days with losses exceeding VaR, p is the expected failure rate $(1-\alpha)$, and f is the actual failure rate (x/n). The results are compared against the critical value of 3.841 to determine whether to reject the null hypothesis. If $LR < 3.841$, the null hypothesis is accepted, indicating the validity of the optimized parameters. (4) Set traditional VaR parameters (α=95%, t=1; α=99%, t=10) as the control group and the optimized parameter combination as the experimental group. Compare their tail risk capture capability, spectral risk measure values, and backtesting failure rates to validate the superiority of the optimized parameter combination. (5) Compare parameter differences under varying risk aversion levels λ to validate the model's applicability to different risk preference choices.

5. CONCLUSIONS

This paper constructs an optimized VaR parameter selection model based on spectral risk measures. Empirical testing yields the following core conclusions:

(1) The traditional method of selecting parameters based on normal distribution has been found to be limited. As shown in Table 1, the actual results tell us that the skewness of the daily return series of CSI 300 index is 1.0078, the kurtosis is 22.3392, the Jarque-Bera statistic is very large (18,911.16), and the p value is almost zero. This clearly rejects the

assumption that the data is normal distribution, which is in line with the theoretical basis of "the return of financial assets is non-normal". It shows that the return of financial assets shows obvious characteristics of thick tail and right deviation. Traditional parameter selection methods can not adapt to this distribution, while spectral risk measurement based on historical simulation can effectively adapt to them.

Table 1. Descriptive Statistics of Daily Returns for the CSI 300 Index

Statistical Indicators	Numerical Value	Explanation of Indicator Meanings
Sample Size	1212	Number of observed daily effective yield values
Mean Daily Return (%)	0.0068	Average daily yield approaches zero, consistent with market norms
Standard Deviation of Daily Returns (%)	1.3705	Moderate yield volatility
Skewness	1.0078	Right-skewed distribution with higher probability of extreme positive returns
Kurtosis (Overall)	22.3392	Significantly greater than 3, exhibiting pronounced peaked characteristics
Jarque-Bera Statistic	18911.1576	Core statistical measure for testing normal distribution
JB Test p-value	0.000000	$p<0.05$, rejecting the hypothesis of normal distribution
Normal Distribution Test ($\alpha=0.05$)	no	Confirming returns follow a non-normal, peaked distribution with heavy tails

(2) Spectral risk measures can well meet the actual needs of enterprises with different risk preferences. As shown in Table 2, the experimental results show that under the three risk aversion degrees, the optimal confidence level is always 0.95, and the optimal holding period is one day. This shows that a shorter holding period (one day) is more in line with the liquidity characteristics of CSI 300 Index, and a confidence level of 0.95 can balance risk capture and capital efficiency. In addition, the spectral risk measures calculated by the optimized parameters are usually significantly lower than the traditional empirical parameters, which shows that spectral risk measures can give consideration to both risk preference and the accuracy of risk measurement.

Table 2. Optimal VaR Parameters and Spectral Risk Measures under Different Risk Preferences

Risk Aversion Level	Risk Aversion Coefficient λ	Optimal Confidence Level α	Minimum Spectral Risk Measure (SRM)	Backtesting Failure Rate (%)	Expected Failure Rate (%)	Constraint Satisfaction Status
High risk aversion	0.1	0.950	0.024633	4.95	5.00	yes

Table 2. (continued) Optimal VaR Parameters and Spectral Risk Measures under Different Risk Preferences

Risk Aversion Level	Risk Aversion Coefficient λ	Optimal Confidence Level α	Minimum Spectral Risk Measure (SRM)	Backtesting Failure Rate (%)	Expected Failure Rate (%)	Constraint Satisfaction Status
Moderate risk aversion	0.5	0.950	0.024148	4.95	5.00	yes
Low risk aversion	1.0	0.950	0.024088	4.95	5.00	yes

(3) Spectral risk measurement can improve the ability of VaR parameter selection to capture tail risk. As shown in Table 3, the back test results show that the failure rate of the optimized parameters all meets the condition of $|f-(1-\alpha)| \leq 0.02$, and the LR value has always been below the critical threshold of 3.841. In all cases, the original assumptions are accepted, which proves that the optimization parameters are effective. Compared with the traditional empirical parameter 2 (α=99%, t=10), the spectral risk measure corresponding to the optimized parameter is reduced by 65.24%. This shows that the optimized parameters show SRM, verify the effectiveness of the optimal empirical parameters, and bring better risk measurement performance.

Table 3. Comparison of Optimized Parameters and Traditional Empirical Parameters

Parameter Type	Confidence Level α	Holding Period t (days)	Spectral Risk Measure (SRM)	Backtesting Failure Rate (%)	Reduction in SRM Compared to Traditional Parameter 2 (%)
Optimized parameter (λ=0.1)	0.950	1	0.024633	4.95	65.24%
Traditional parameter 1 (empirical value)	0.950	1	0.024633	4.95	65.24%
Traditional parameter 2 (empirical value)	0.990	10	0.070892	0.85	—

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