From Time to Topology: A Comparative Review of Statistical Rigor and Relational Memory in Graph-Based Financial Forecasting

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ABSTRACT

This paper discusses how the concept of financial forecasting changes. He compared the "linear lag operator" of traditional econometrics with the new concept of "graph-based learning". Traditional methods are very strict with statistics, but books and articles say that they do not see the "nonlinear dependence" of the high-frequency market well. On the other hand, "Connectionist" models can predict well, but we don't know much about their structures. By creating a "topological bridge" with "visibility graph", this paper puts forward a theoretical concept. In this concept, "market memory" is no longer regarded as the proximity of time, but as "geometric connectivity". This analysis shows that "GNN" based on visibility structure may provide the advantages of "glass box". This may help to keep the interpretable artificial intelligence (XAI) robust in the face of change (instability). This paper concludes that it is a necessary theoretical development to turn to "topology" representation. This is crucial for promoting a sovereign and independent financial system.

Keywords: GARCH; Graph-Based Learning; Neural Networks; Explainable AI

1. INTRODUCTION

1.1 Background and Motivation: The High-Frequency Shift

High frequency trading (HFT) is a very important part of algorithmic trading. It uses a very powerful computer to execute a large number of orders at a very fast speed (usually within a fraction of a second). Our goal is to take advantage of small differences in the market. In the high-frequency environment, people often say in books that the traditional econometric framework is not good enough. They can't capture the microstructure changes and very rapid discontinuities that often occur in today's price changes. With more and more market participants using autonomous systems driven by artificial intelligence to execute orders, the theoretical definition of "market memory" is changing completely. In this case, market memory is the duration of past price dynamics and its influence on future fluctuations. This is the mechanism that the historical structure model helps to make the short-term prediction signal more accurate. The existing research shows that we are turning to complex multidimensional state representation. This shows that a wider framework is needed to adapt to the nonlinear reality of modern financial infrastructure.

1.2 Research Gap: Rigor vs. Flexibility

At present, the literature has determined the strong differences of financial modeling paradigm. The "classical paradigm" is recognized for its statistical rigor and inherent stability; However, people often say that it lacks the flexibility needed to adapt to long-term dependence and chaotic market dynamics. On the contrary, the "Connectionism Paradigm" driven by deep learning architecture shows remarkable predictive ability, but it usually operates as a non-static "black box", which brings problems of transparency and theoretical basis. This analysis points out a significant lack of research: the lack of a comprehensive framework to reconcile the flexibility of the algorithm with the structural interpretability required by institutional and regulatory standards.

1.3 Objectives and Scope

In this paper, a theoretical transition from time series to topological representation is proposed to fill this gap. By examining and comparing the different definitions of memory, this paper evaluates the effectiveness of graph-based learning as a potential synthesis of statistical rigor and predictive ability. The research objectives are defined as follows:

(1) To analyze the structural limitations of Euclidean lag operators within non-stationary financial environments.

(2) To examine the Visibility Mechanism as a conceptual non-linear filter for market noise.

(3) To evaluate the "Topological Bridge" as a theoretically superior, interpretable architecture for contemporary financial infrastructure.

2. RELATED WORK / LITERATURE REVIEW

2.1 The Classical Paradigm: Linear Temporal Lags and Statistical Rigor

The basic framework of forecasting money is based on Euclid's viewpoint, which imagines market memory as a function that decreases with time. This model was created by Robert Fry Engle [1] and Autoregressive Conditional Heteroskedasticity (ARCH) model. He believes that the indicators used for forecasting are within a fixed time range. Subsequently, Tim Bollerslev [2] improved this by Generalized ARCH(GARCH), and made the memory more persistent by adding the past variance prediction. These methods have become the industry standard of statistical rigor and stability of risk reporting.

However, many studies show that linear lag may not be enough to describe the operation mode of modern market. Then, we find the characteristics of long-term memory, in which the return is much slower than that predicted by autoregressive model. This means that time distance is not a good way to measure relationship dependence [3]. In addition, the market shows complex nonlinear chaotic structures, which are usually similar to random noise when conducting linear tests [4]. These criticisms point out an important limitation: it is impossible to capture the multidimensional nonlinear dependence of systematic market changes by relying on time series.

2.2 The Connectionist Paradigm: Algorithmic Flexibility and the Black-Box Dilemma

The limitation of lag-based linear model promotes the "Connectionism" paradigm. This model uses deep learning to capture high-dimensional market models. The core of this change is the long-term and short-term memory (LSTM) architecture [5]. It uses dynamic gating mechanism to replace fixed lag. LSTM selectively retains or discards information through its internal state, which brings the algorithm flexibility needed to deal with the long-term dependence completely ignored by GARCH framework. In the high frequency trading (HFT) environment, these architectures successfully simulate the spatio-temporal flow and discontinuity, which are usually not realized by traditional linear filters [6].

Although it has strong forecasting ability, this model often brings statistical transparency problems. Modern frameworks usually need well-designed preprocessing pipelines to clean up non-static data [7]. This dependence on "clean-up" of external sources reveals structural fragility: although connectionist models are highly adaptable, they usually lack the static constraints inherent in classical econometrics. In addition, these models are usually described as providing a limited mathematical description of the causal relationship of specific predictions. This lack of structural interpretability constitutes a potential regulatory risk, because the internal weight cannot be easily transferred to the economic framework needed to manage institutional risks.

2.3 The Emergence of Graph-Based Learning: From Sequences to Topologies

The latest development of financial forecasting means a change: we no longer regard time series as linear series, but model them as complex nonlinear networks. This topological change is introduced with the introduction of visibility graph [8]. This method transforms time series data into network, while retaining structural attributes such as versatility and long-term memory. Different from GARCH's fixed offset or LSTM's opaque gate, visibility graph redefines memory through topological connection, which makes it easier to realize geometric representation of market dynamics. This method can capture the "line of sight" between distant market events, thus effectively bypassing the limitation of Euclidean time distance.

Graph Neural Network, (GNN) is a special deep learning model. It is used to process data similar to graphics (a network of connected elements). It understands complex relationships by repeatedly transmitting and grouping information between connected data points. Based on this structure, the price chart uses graphical neural network (GNN) to capture the relationship dependence between asset movements [9]. This method retains the predictive ability of deep learning, but fixes it in an understandable spatial framework. By adding lateral information, graph-based learning is more than just predicting a single variable. It provides a multi-faceted perspective. This makes the ability of the algorithm match the structural reality of the global financial market.

3. THE RECONSTRUCTION OF MEMORY: FROM TIME TO TOPOLOGY

3.1 The Geometry of Information: Lags vs. Links

To evaluate the reconstruction of memory, one must contrast the mathematical operators governing information processing. The classical paradigm, exemplified by the GARCH (1,1) framework [2], defines conditional variance (σ_t^2) through a linear lag operator (L):

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{1}$$

This structure imposes Euclidean time constraints, under which market memory is modeled as the weighted and decreasing sum of its direct time series. This architecture assumes a unified information flow, in which remote structural events are systematically attenuated by attenuation parameters (α, β), regardless of their qualitative importance.

On the contrary, the shift to graph-based learning redefines memory as topological connection rather than chronological proximity. The relationship between price points is captured by a random matrix (A), which serves as a non-local map of the system state [9]. This matrix is not a sliding window observed recently, but encodes the network as a set of binary relationships, in which the connection between two distant points only exists when they remain "visible" to each other in the market geometry. By bypassing the requirement of temporal adjacency, this operator helps to maintain dependencies for a long time.

3.2 The Visibility Mechanism: A Non-Linear Filter for Market Noise

The conversion of a 1D financial time series into the Adjacency Matrix is predicated on the Visibility Criterion. As formalized two discrete data points (t_a, y_a) and (t_b, y_b) are considered topologically connected if, for every intermediate point (t_c, y_c) such that $t_a < t_c < t_b$, the following linear inequality is satisfied [8]:

$$y_c < y_b + (y_a - y_b)\left(\frac{t_b - t_c}{t_b - t_a}\right) \tag{2}$$

This standard is used as a nonlinear geometric filter. Traditional econometrics regards noise as a smooth random variable, while visibility algorithm uses relative height and price fluctuation distance to determine connectivity. Small price fluctuation can not block the "line of sight" between the vertices of the main structure, thus ensuring that the obtained network retains the multi-faceted nature of the original series.

Through this mechanism, long-term dependency is re-coded as Closeness Centrality. For any node i, centrality (C_i) is defined as:

$$C(i) = \frac{n-1}{\sum_{j=1}^{n} d(i,j)} \tag{3}$$

In the visibility graph, the historical price point with strong centrality is described as an event that can reach the rest of the network in the smallest topological step. These nodes have become market landmarks-events described as structurally important events in the literature, although the time distance is far away, are still close to the present in topology. By shifting the focus of analysis from data historicity to its topological centrality, the proposed framework obtains high-resolution memory of important historical changes that the linear model may disappear due to lag.

Therefore, the generated network retains the multi-faceted nature of the series. Although the standard LSTM treats the input as a unified stream, the visibility mechanism effectively resamples the data according to the structural importance. High volatility events become the center with high centrality, while low volatility events are still peripheral in topology. This shows that the reconstruction of market memory is relatively invariant to small-time disturbances, but focuses on the geometric skeleton of market evolution.

4. EVALUATION OF STATISTICAL RIGOR IN NEURAL ARCHITECTURES

4.1 The Crisis of Non-Stationarity: Spurious Forecasts and the Topological Bridge

One of the main problems in financial forecasting is the instability of price data. Their statistics are constantly changing, and we can't predict how. It is mentioned in the book that traditional neural networks, especially LSTM, often make "stability fallacies". It is believed that the model learned during the period of great price changes will also run in the quiet period. This difference may lead to wrong prediction. The model is too accustomed to the small fluctuation of local prices, and takes random noise as the real market rule [10].

However, the existing research puts forward a methodological solution, that is, shifting from time domain to network domain [11]. As suggested, the concept of "topological bridge" shows that although the original price level is naturally non-stationary, the geometric properties of its related visibility graph often show structural stability. Specifically, the degree distribution (describing the connectivity characteristics of price points) tends to be stable, even in the change of market system. Using visibility graph as a bridge, the proposed framework effectively transforms chaotic linear sequences into static cyberspace. This makes it possible for the graphic neural network (GNN) to maintain its structural integrity. When the value fluctuates, the "shape" of market memory remains consistent, allowing the architecture to fix the forecast on the stable topological characteristics, rather than the peak of price fluctuation.

4.2 Persistence vs. Adaptation: Measuring Structural Integrity

To evaluate statistical rigor, it is necessary to distinguish between true persistence and stochastic adaptability. A naive model can simply adapt to the latest price fluctuations and produce a forecast as a delayed response to the past. In order to ensure that GNN captures the continuous trend, the analysis suggests to go beyond the mean square error (MSE) and use the product sum degree distribution stability of topology.

The content of Visibility Graph describes the complexity of this structure. A framework that captures real persistence should generate a graph with stability distribution, where $P(k) \sim k^{-\gamma}$. Although the γ index may change during regime change, strict architecture is characterized by constant topological substances, even if the original price values are different. This shows that the model captures the potential proportional law rather than misleading local noise.

In addition, it is suggested to measure the adaptive delay, which is the time required for the central hub of the graph to be reconfigured after the structural fracture. The powerful framework maintains a stable topological core, and it will change its connection only when the multi-factor analysis confirms the fundamental change of the long-term memory of the market. This two-level verification ensures that memory reconstruction is a strict statistical representation of market risk, not just a calculation artifact.

5. XAI AND STRUCTURAL INFORMATION IN FINTECH DECISION-MAKING

5.1 Synthesis: The Convergence of Geometry and Alpha

The integration of visibility graph and graphical neural network (GNN) represents a change of thinking: from simple numerical prediction to market geometric analysis. As we have seen in [12], the frontier of financial technology is increasingly defined by the architecture that can capture the structural characteristics of market dynamics. This change is proved by the adoption of terrain data analysis (TDA) and alternative data sets by the quantitative fund of the organization.

The existing research shows that non-Euclidean learning is becoming more and more useful in high frequency trading (HFT) environment. In these environments, when the change is very fast, the traditional linear model may not work properly [11]. Using topological bridge, this paper shows that market memory can be reconstructed into a stable relational network. In practice, this matches the structural alpha extraction: the prediction advantage comes from the persistent geometry of price changes, not the noise of time. The literature about using TDA shows that it has better performance in identifying system anomalies (such as "Flash crash"). In these cases, the graphic topology usually shows a fracture before the price changes significantly. In this case, risk is no longer regarded as a hidden variance parameter, but as a visible topological decline.

5.2 Interpretability and the "Glass Box" Logic

The natural opacity of deep learning architecture is still the main problem of compliance rules, especially the "exploration right" framework of GDPR. However, the visibility diagram method provides the potential advantage of "glass box" by providing a repeatable audit trail of market memory. As explained in article [13], an effective financial interpretable AI(XAI) must be able to relate the output of the model to a specific time precursor.

In the GNN-Visibility framework, practitioners can perform Centrality Audit to interpret the signals of the model. For example, a Sell signal generated during a moderate fluctuation can be analyzed by the following logical increment:

(1) Identification: The auditor identifies that the present node shares a high-weight edge with a node representing a historical period of systemic stress, such as the 2008 Financial Crisis.

(2) Contextualization: Given that the Visibility Criterion requires a direct "line of sight", this connection implies that the current geometric profile of the market structurally mirrors the historical regime.

(3) Verification: The model identifies a Topological Isomorphism—a structural similarity in market "shape" that transcends chronological distance.

This method transforms artificial intelligence from opaque prediction engine to relational map. This allows the algorithmic reasoning to be verified by comparing it with valid historical precedents. This match between mathematical rigor and human intuition provides the transparency required by the current autonomous financial standards.

5.3 Future Directions: Towards Sovereign Multi-Graph Systems

This analysis focuses on the visibility of discrete time series, but the theoretical frontier is the construction of high-dimensional multi-graphs. In this state, every asset in the global wallet is like a node in a larger "network". This has opened up several important research approaches:

(1) Cross-asset topology synchronization: Future models can use cross-visibility diagrams to draw the relationships between different asset classes. Measuring the synchronization of these graphs may help to find hidden infection channels.

(2) Transition to sovereign distributed state machine: In the context of decentralized infrastructure, visibility map can be used as a map of system state history. Integrating these topologies into decentralized ledgers can promote sovereign intelligence, in which market risk is calculated locally by the network.

(3) Dynamic Graph Rewiring for real-time adaptation: The last research direction is to switch from static adjacency matrix to dynamic graph rewiring. By combining the trend detection function with real-time GNN, the system can dynamically delete or add links when market conditions change. This will enable the model to adjust its memory structure in real time to match the current volatility, and perhaps bridge the gap between mathematical persistence and market reality.

6. CONCLUSION AND FUTURE WORK

6.1 Summary of Findings: Memory as Relationship

The main contribution of this research is to rethink market memory in theory under the background of contemporary financial technology. From Euclidean label to topological connectivity, this paper shows that memory in complex financial systems can be defined more accurately as a set of relational dependencies, rather than strict time order. Synthetic literature shows that market information will not decrease linearly with time. On the contrary, it keeps a geometric line of sight, pointing to historical events with structural significance. The proposed topological bridge is beneficial to the stable representation of information and seems to resist the crisis of nonstationnarity.

6.2 Limitations: The Cost of Complexity

Despite the established theoretical advantages, the implementation of this method involves significant practical limitations. The main limitations are computational overhead and data granularity. Converting high-frequency time series into visibility map may cause delay in real-time execution in high-frequency environment. In addition, although the graph-based method captures market dynamics through geometric abstraction, it is still a mathematical representation of human behavior. Although the framework can detect structural fracture, it has no ability to consider qualitative exogenous shocks or geopolitical changes. Therefore, as a complex mapping tool, the architecture needs human institutional supervision to explain the qualitative drivers of topology change.

6.3 Future Work: The Rise of Statistically Informed GNNs

The future research road lies in developing hybrid models, especially statistical infographic neural networks. Instead of just choosing between GARCH's statistical rigor and GNN's algorithm flexibility, FinTech's next development should combine these methods into a dual-process system. Future research should explore "attention-based preprocessing", in which real-time multifunctional detection parameters are used to decide whether to strengthen or break links in visibility graphs. These advances will help to better match mathematical persistence with the changing reality of global financial markets.

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