Derivatives Pricing: Black-Scholes, Real Options, and Market Completeness

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ABSTRACT

Options and derivatives pricing have produced some of financial economics' most elegant theoretical achievements, most consequential practical applications, and most spectacular market failures. Black, Scholes (1973) and Merton's (1973) option pricing formula—published simultaneously in the Journal of Political Economy and the Bell Journal of Economics—established a no-arbitrage pricing framework that earned Scholes and Merton the 1997 Nobel Prize in Economics and gave practitioners a widely used tool for pricing options, hedging risk, and valuing financial guarantees. The model's assumptions—constant volatility, log-normal returns, continuous trading—have been systematically relaxed to address the 'volatility smile' (implied volatility varying with strike price), heavy-tailed return distributions, and stochastic volatility that actual markets exhibit. Real options analysis—applying option pricing theory to capital investment decisions—has extended derivative theory to the valuation of strategic flexibility in corporate investment. This paper reviews option pricing theory, the Black-Scholes formula and its limitations, stochastic volatility models, and real options applications.

Keywords: Black-Scholes; Options Pricing; Real Options; Volatility Smile; Stochastic Volatility; Derivatives; Merton; Capital Budgeting.

1. INTRODUCTION

The pricing of contingent claims—contracts whose payoff depends on the realized value of some underlying asset—presented a fundamental theoretical challenge before Black, Scholes, and Merton's breakthrough: options have complex, non-linear payoffs whose fair value depends on the unknown future distribution of the underlying asset's price. Black and Scholes' (1973) key insight was that a continuously rebalanced portfolio of the underlying asset and a risk-free bond could replicate any option's payoff exactly—making the option's fair price equal to the cost of this replicating portfolio, which can be computed without any information about investors' risk preferences or the expected return on the underlying asset. This preference-free pricing—the most important feature of the Black-Scholes framework—made derivative pricing as objective as bond pricing and enabled the explosion of financial engineering that followed.

2. THE BLACK-SCHOLES-MERTON MODEL

2.1 Model Assumptions and Formula

The Black-Scholes-Merton (BSM) model assumes: (1) the underlying asset price follows geometric Brownian motion with constant drift and volatility; (2) no dividends during the option's life; (3) markets are frictionless (no taxes, transaction costs, or short-selling restrictions); (4) the risk-free rate is constant; (5) trading is continuous. Under these assumptions, the price of a European call option is: $C = SN(d_1) - Ke^{(-rT)}N(d_2)$, where S is the stock price, K is the strike price, T is time to expiration, r is the risk-free rate, σ is volatility, and $d_1 = [\ln(S/K) + (r + \sigma^2/2)T]/(\sigma\sqrt{T})$, $d_2 = d_1 - \sigma\sqrt{T}$. The formula's elegance lies in what it does not require: no assumption about the expected return on the stock, no information about investor risk preferences.

2.2 The Volatility Smile

The BSM model's assumption of constant volatility is violated by financial market data: options at different strike prices for the same underlying asset and expiration date show different implied volatilities—the 'volatility smile' or 'volatility skew.' Equity options typically show a 'volatility skew' with higher implied volatility at lower strikes (put options), reflecting negative skewness and excess kurtosis in equity returns (crashes are more common than the log-normal distribution predicts). This pattern, absent before the 1987 market crash, emerged after that event as market participants updated their models to account for fat tails and crash risk. Stochastic volatility models (Heston, 1993) and jump-diffusion models (Merton, 1976) have been developed to fit the volatility surface, at the cost of the closed-form analytic tractability of the original BSM formula.

3. REAL OPTIONS

Table 1. Real option types and corporate examples.

Real Option Type	Description	Corporate Example
Option to defer	Wait for better information before investing (call option on project)	Mining company delaying mine development; pharmaceutical awaiting trial results
Option to expand	If initial investment succeeds, expand (call option on additional scale)	Retail chain entering new market; pilot project preceding rollout
Option to abandon	Exit project if value falls below salvage (put option on continuation)	Flexible manufacturing; modular plant design with exit rights
Option to switch	Switch between inputs or outputs as relative prices change	Dual-fuel power plants; flexible manufacturing systems

4. REAL OPTIONS IN PRACTICE

Dixit and Pindyck’s (1994) ‘Investment Under Uncertainty’ (Princeton University Press) systematized real options analysis for corporate investment decisions, demonstrating that the standard NPV rule systematically undervalues investments when firms have flexibility options—particularly the option to defer irreversible investments until better information arrives. The real options framework makes explicit what NPV ignores: the value of flexibility increases with uncertainty (more volatile environments make waiting more valuable) and with irreversibility (more irreversible investments make the option to defer more costly to exercise and therefore more valuable). Graham and Harvey (2001) in the Journal of Financial Economics document that while real options theory is well-established academically, only about 27% of CFOs use real options analysis formally in capital budgeting—suggesting a significant gap between theory and practice driven by implementation complexity.

5. CONCLUSION

Option pricing theory represents one of financial economics’ greatest theoretical achievements, providing a no-arbitrage pricing framework that is both analytically tractable and practically implementable. The Black-Scholes model’s preference-free pricing insight enabled the explosion of derivative markets that now facilitate risk transfer across the global financial system. The model’s limitations—particularly its failure to account for the volatility smile—have motivated substantial model extensions that better fit empirical option price patterns at the cost of analytic tractability. Real options analysis extends derivative theory to strategic corporate investment, providing a framework for valuing flexibility that standard NPV analysis ignores. Future research should examine how machine learning models can fit the volatility surface more accurately than parametric stochastic volatility models, and whether the behavioral biases documented in equity markets produce systematic mispricings in derivatives markets.

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