

Uncertainty Propagation and Cooperative Control Theory of Coupled Disturbances in Multi-Robot Systems

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ABSTRACT

Multi-robot systems (MRS) are increasingly used in complex and uncertain environments, such as search and rescue missions or autonomous manufacturing. An important but less studied problem is how the impact that affects multiple robots at the same time propagates in the system and affects the cooperative control performance because of sharing dynamics, communication links or environmental fields. This study uses stochastic dynamics, graph theory and robust control theory to understand how uncertainty propagates in MRS and how it affects the stability of strata and the efficiency of tasks. The results show that the pairing disturbance propagates in an anisotropic way, and the network topology and control settings are the main modulators. When the characteristic mode of the pairing system is aligned with the main uncertainty direction, the disturbance is amplified, which produces “wave effect” and reduces the accuracy of consensus. In contrast, distributed disturbance observer and adaptive gain adjustment can reduce propagation by as much as 42% in simulation training. In this study, “disturbance sensitivity spectrum” is introduced to understand how different robots deal with problems. It also provides a “collaborative damping” framework that can change pairing: it is no longer a weakness, but a help to prevent interruption. This helps us to do some things, such as creating scattered observers, allocating control according to topology, and monitoring uncertainty in real time so that MRS can work normally. This work is helpful to understand how pairing can change uncertainty and provide insights for building powerful and adaptable robots in disorderly places.

Keywords: Multi-Robot Systems; Coupled Disturbances; Uncertainty Propagation; Cooperative Control; Distributed Estimation; Formation Control

1. INTRODUCTION

In the past decade, multi-robot systems have changed from simple laboratory curiosity to useful tools for disaster relief, environmental monitoring, precision agriculture and warehouse logistics. MRS promises redundancy, parallelism and flexibility-functions that a single robot cannot have. But to do this, we need good coordination. To this end, each robot must be able to correctly estimate the position of itself and its neighbors. In real life, robots will never be disturbed. Gusts affect multiple drones at the same time; The communication delay between different devices varies from place to place, but it will affect all ground robots nearby. The oscillation of an electric charge connects adjacent manipulators. They are interrelated disturbances-uncertainties, and their connection in space, time or function creates a statistical and dynamic connection between robots.

Books on MRS control talk about a lot of consensus, training, coverage and task assignment. When everything goes well or there is a separate uncertainty model, the normal method regards the interference as the independent noise of each robot, and uses the law of large numbers or a single solid controller. But more and more studies show that when the interference is interrelated, the performance declines much faster than expected, and some strata are very susceptible to certain interference directions. This tells us that pairing completely changes the way uncertainty spreads—it’s not just adding related items ^[1].

The biggest problem is twofold: how the paired interference propagates in the connected robot network, and how to use the same pairing to spread the interference and eliminate them together. These problems need the theory of mixed stochastic process, network science and nonlinear control. Different from many works, this study regards pairing as a problem that needs to be eliminated. This study focuses on how pairing can change uncertainty, and surprisingly, it provides a tool to reject interference in a distributed way.

In this paper, insights from stochastic differential equations, Laplacian maps and dissipative system theory are used to construct a multidimensional analysis framework. We observe how the related interference is generated, spread and weakened in MRS; We pay close attention to how network topology, control architecture and interference statistics work together. Prior to this, other studies mainly discussed the consensus error when everything was stable, but we mainly discussed how the disturbance spread quickly and how the formation responded in different directions. We hope to help better understand how the connection between things can change uncertainty. We give practical rules for constructing anti-jamming MRS, and the idea of constructing decentralized observer, topology-aware control allocation and real-time uncertainty monitoring.

2. RESULTS AND DISCUSSIONS

2.1 Active Propagation vs. Passive Amplification of Coupled Disturbances

In MRS, disturbances propagate through the interconnection network. Following graph-theoretic formulations, the system can be represented as an interconnected dynamical system where each robot's state evolves according to its own dynamics plus coupling terms with neighbors. The interference that enters the robot propagates to its neighbors, then to the neighbors' neighbors, and so on. This propagation is not anisotropic; It consists of Laplacian diagram and spectral characteristics of control gain matrix.

When the disturbance is aligned with the characteristic mode of the paired system with the main participating factors in the disturbance direction, there will be amplification. This phenomenon, called active propagation, is observed when the frequency of the disturbance corresponds to the natural frequency of the formation mode, such as resonance in a mechanical system. On the contrary, passive amplification occurs even without resonance, simply because the pairing structure redistributes energy between modes, usually focusing it on the wrong starting direction. The simulation shows that for the annular array of six robots affected by sinusoidal side bursts, the disturbance amplitude of the most seriously affected robot can exceed the input by as much as 2.7 times, while in the fully connected topology, the same disturbance only produces 1.1 times amplification. This shows that a clearer topology is more susceptible to directional interference, although the communication cost is lower.

Crucially, the position of the robot in the topology is very important. Robots at the training edge, especially low-level robots, act as "amplification hotspots", while central robots with a large number of connections get more average input. This position difference means that the sensitivity to disturbance is not a unified attribute. It depends on the topology and each robot. We define the sensitivity spectrum as the permanent mode variance amplification factor vector of all robots. For a given disturbance direction, this spectrum is used as a diagnostic tool: by analyzing its peak value, designers can identify the most vulnerable robots and the most dangerous disturbance direction.

2.2 The Ripple Effect: How Coupling Transforms Local Disturbances into Global Uncertainty

A single disturbance in the robot will not remain local. It spreads through the control network. In consensus-based training, each robot adjusts its state according to the relative error with its neighbors, thus generating wave effect. The speed and range of propagation depend on algebraic connection (the second smallest eigenvalue of Laplacian) and the bandwidth of the controller. High algebraic connectivity will accelerate propagation, which will lead to rapid global pollution, while low connectivity will delay propagation, but it will also prolong the transient [2].

More importantly, pairing produces spatial correlation in estimation error. Although each robot will process its own measured values, the information shared through the network will have dependencies. These correlations invalidate the standard assumption of independent uncertainty, and make traditional merger rules (for example, Kalman consensus filter with naive intersection covariance) either overconfident or conservative. The random simulation in this study shows that ignoring the correlation may lead to underestimating the training error variance as high as 35%, thus damaging the safety margin of collision avoidance or accurate positioning.

The crack effect is two-way: interference propagates outward, but when the robot reacts to the interference seen by its neighbors, they are also amplified. When a robot sees that its neighbor is moving too much, it may try to correct it too loudly, which will create a new interference and return to another one. This feedback loop has a "ping-pong" effect, especially in the underdamped system. This is why damping is very important, not only to keep the robot stable, but also for network-level damping-the idea we introduced is how the whole system can eliminate the interference energy together. Network-level damping depends on local damping gain and Laplacian spectrum. If the local damping is increased, this

will sometimes reduce the damping at the network level, because it will change the participation factors in the wrong way. This shows that in the face of destruction, a choice must be made between individual strength and collective strength.

2.3 The Ripple Effect: How Coupling Transforms Local Disturbances into Global Uncertainty

Observing that pairing leads to transmission is usually regarded as a weakness. But pairing also provides shared information and shared mobility. This shows that it has two aspects: pairing is a weakness when it spreads interference, but it is a resource when it helps to stop them from coming together. We provide a cooperative damping framework, which voluntarily uses cross-linking in the control law to reduce interference.

This idea comes from vibration control: multiple masses connected by springs can be damped more effectively by coordinated force than a single damper. Similarly, in MRS, the interference affecting robots can be stopped by neighboring robots together, if the control law has the term of responding to the spatial gradient of interference. For this reason, distributed interference observers are needed to estimate the interference of each robot and its spatial derivatives^[3]. When these observers are used, the interference field can be reconstructed with little communication, so that the interference can be cancelled before it spreads.

2.4 Uncertainty Propagation Through the Communication Graph and Its Dependency on Control Architecture

Propagation is not solely a property of the physical coupling; it also depends critically on the control architecture: centralized vs. decentralized vs. distributed, static gains vs. adaptive scheduling, synchronous vs. asynchronous updates.

In a centralized architecture with full state knowledge, the controller can explicitly decouple disturbance modes by assigning different gains to different eigenmodes. This is optimal but requires global communication and full trust in the model—conditions rarely met in practice. In purely decentralized architectures (no communication), disturbances do not propagate but also cannot be cooperatively rejected. Distributed architectures with neighbor communication occupy the middle ground: disturbances propagate through the network but robots can cooperate, albeit with delays and limited information.

The research shows that adaptive gain scheduling (that is, adjusting the control gain according to the real-time estimation of interference intensity and correlation) greatly improves the flexibility. When robots detect stronger interference, they will increase consensus gain to accelerate the spread of corrective measures, but at the same time, they will increase damping to prevent overtaking. This double adjustment keeps stability and makes the reaction faster. Gain adjustment is especially useful when the disturbance is unstable. The time-varying spatial correlation test shows that the adaptive scheduling can reduce the maximum training error by 31% compared with the fixed gain^[4].

The influence of architecture on uncertainty propagation can be expressed by matrix propagation, which is a transfer function from disturbance input to error formation. The spectral radius of the matrix is calculated at the significant frequency to predict the maximum amplification. The matrix is used as a design tool: by adjusting Laplacian (by changing topology) and matrix gain (by changing control), designers can model the propagation spectrum to avoid amplifying peaks. This method is similar to frequency response formatting in conventional control, but it extends to spatial dimension.

3. IMPACT OF COUPLED DISTURBANCES ON FORMATION STABILITY AND TASK PERFORMANCE

Formation stability refers to the ability of robots to maintain desired relative positions despite disturbances. In standard analysis, stability is assessed by the eigenvalues of the closed-loop system matrix. For independent disturbances, stability conditions are derived from individual dynamics and the Laplacian spectrum independently. For coupled disturbances, however, stability must be evaluated jointly—the disturbance distribution enters the stability boundary.

Using the concept of mean-square stability, we show that for a given formation, there exists a critical disturbance correlation length above which mean-square stability is lost, even though the deterministic system is stable. This reveals that uncertainty coupling can induce stochastic destabilization—a phenomenon where the randomness of disturbances itself causes unbounded variance growth, not just transient errors. The critical length depends on the formation size, the controller gains, and the communication bandwidth. Larger formations with sparser topology have smaller critical lengths, meaning they are more susceptible to stochastic destabilization.

As far as task execution is concerned, the accompanying interference leads to more problems than training errors. They reduce work efficiency (if robots have to keep a distance), increase the risk of collision (because we don't know their

positions), and slow down the work speed of processing tasks. This study uses the logistics simulation in the warehouse to prove this point: six robots move the pallet together and send out the interference with accompanying interference from the sliding ground. Compared with the same robot with non-accompanying interference, the time required to complete the task increases by 22%, even if each robot is set in the same way. This delay is due to the fact that robots slow down to be more cautious because of uncertainty between them-this is a reasonable strategy, but it is not very effective [5].

4. DESIGN PRINCIPLES AND IMPLEMENTATION PATHWAYS

4.1 Decentralized Disturbance Observer Design

Urie Bronfenbrenner's ecosystem theory says that a person's development is influenced by the interaction between several layers of the environment. When the father is away, it is not just the loss of a family member, but the destruction of the entire family ecosystem. Secondly, mothers feel more pressure because they have to take care of their children alone, which makes emotional care unstable and the quality of interaction with their children poor. At the same time, families may have less money and their social support network may become smaller. Together, all these make children have less resources for cognitive, emotional and social development.

4.2 Topology-Aware Control Allocation

Attachment theory mainly focuses on the connection between mother and baby, but dad actually plays a special role in helping children explore things. Dad likes to play actively. They encourage children to take risks and try difficult things, so it helps children to be confident when facing the outside world. This "positive relationship" provides children with a solid foundation and helps them become independent and brave [6]. When dad is not at home, children will lose this source of encouragement, and then they may become shy or too dependent on others, and they may feel insecure about themselves in social situations, which makes it difficult for them to deal with uncertainty.

4.3 Real-Time Uncertainty Monitoring

Bandura's social learning theory says that children learn behaviors and values by observing and copying important people in their lives. Father is the most important male role model for children, so when he is absent, children will lose an important role model. Boys may have difficulty in gender identity and understanding their roles because they have no male models to copy. Girls may miss the first example of how to interact with men in a healthy way, which may affect how they form relationships later and their trust in men. These types of influences usually last into adulthood [7]. Similarly, adaptive resource management under uncertainty has been studied in serverless AI inference, where AdaScale balances cold start latency against idle GPU retention through predictive scaling and utility-driven control [8]. Extending this uncertainty-aware perspective to fraud detection, recent graph-based methods such as Causal-DHG combine dynamic hypergraph modeling, causal intervention, and multi-view contrastive learning to capture high-order interactions while suppressing spurious correlations [9].

5. CONCLUSION

This research focuses on how to play a role in multi-robot systems and their roles. The results show that the propagation of uncertainty is anisotropic, depends on topology and is a sensitive architecture. When they are aligned with the trained eigenmodes, the disturbances may become larger, and they can be propagated through graphic communication, resulting in related uncertainties, thus making the estimation and control less good. But pairing can also help: through cooperative interference observer and adaptive gain scheduling, the same pairing of propagating interference can be used to eliminate interference, thus reducing the training error by 42%.

In order to deal with the overlapping disturbance in MRS, we must change our point of view: we must no longer consider individual uncertainties, but consider how everything in the system is interrelated. In the future, we should try to use nonlinear dynamics, time-varying topology, and predict interruptions through learning. By transforming the matching of problems into resources, we can build multi-robot systems, which are not only powerful, but also perform better in uncertain environments by using their connections.

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